The Effect of Internal Scattering on the Color of Fabrics*

GEORGE GOLDFINGER† and JAMES H. WIGGS, *North Carolina State University at Raleigh, School of Textiles, Raleigh, North Carolina 27607*

Synopsis

The effect of internal scattering on the color of dyed fabrics is calculated with the help of a highly simplified model. The predictions agree, at least qualitatively with observation. If an internally scattering fiber is ring-dyed then at lower values of the product of the coefficient of extinction and dye concentration (25–250) it is lighter than a homogenously dyed fiber but at higher values of that product it is darker.

INTRODUCTION

In earlier papers[1,2] a model for an absorbing scattering substrate was presented which permits the prediction of the color of such a substrate from basic optical and geometric properties. Specifically, the color of textile substrates was successfully predicted if the fibers themselves where optically homogeneous. However, fibers are very often optically inhomogeneous; if they are not, socalled delustrants are added which cause internal scattering. This may cause an interesting phenomenon: If the individual fibers are internally nonscattering or only slightly scattering, a commonly occuring nonuniformity of dye distribution, ring-dyeing, may cause the fiber assembly to be lighter in shade than it would be if the same amount of dye were uniformily distributed. If, however, internal scattering is significant, then the same dye distribution will produce under certain conditions a deeper shade than it would were it uniform. The model in question correctly predicts the effect of ring-dyeing on optically homogeneous fibers.[3,4] This paper deals with the entire problem for any level of internal scattering and ringform or uniform dye distribution.

THE MODEL

The original model has recently been extended[5] to account for the influence of fiber separation within the array on the color of a fabric. Those calculations require so much computer time that in this paper it has been decided to use the simpler model, an array in which it is assumed that all fibers are parallel and that they do not influence the reflection-refraction pattern of their neighbors.

It will be assumed that the scattering particles are uniformily distributed and that they are optically and geometrically uniform. It will also be assumed in some cases that the radiation is uniformily scattered around the particles, and in other

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† Present address: Fashion Institute of Technology, Research Department, Room D130, 227 West 27th Street, New York, New York 10001.

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cases that is is scattered only in the "forward" and "backward" direction. It is assumed that errors produced by these inconsistencies cancel.

REFLECTION, REFRACTION, AND ADSORPTION

It will be assumed that the incident radiation of intensity I_0 is normal to the plane in which cylindrical fibers lie. This light beam is subdivided into *INCR* equal increments. Each strikes the fiber at an angle θ (Fig. 1) so that $\sin\theta = d$, from $d = 0$ to $d = r$, the radius of the fiber.

If the continuous medium is air and the index of refraction of the fiber is m then by Fresnel's law the reflections $\rho_\parallel$ and $\rho_\perp$ are given by eqs. (1) and (2):

$$\rho_\parallel = \frac{1}{2}\left(\frac{(1-d^2)^{1/2} - (m^2-d^2)^{1/2}}{(1-d^2)^{1/2} + (m^2-d^2)^{1/2}}\right)^2 \tag{1}$$

$$\rho_\perp = \frac{1}{2}\left(\frac{m^2(1-d^2)^{1/2} - (m^2-d^2)^{1/2}}{m^2(1-d^2)^{1/2} + (m^2-d^2)^{1/2}}\right) \tag{2}$$

and by Snell's law the angle of refraction α is

$$\alpha = \arcsin(d/m) \tag{3}$$

The length of the lightpath lp within the fiber is

$$lp = (2r/m)(m^2 - d^2)^{1/2} \tag{4}$$

and its portion in the core lpc,

$$lpc = (2r/m)[m^2(1-th)^2 - d^2]^{1/2} \tag{5}$$

and its portion through the ring lpr,

$$lpr = lp - lpc$$

$$lpr = (2r/m)\{(m^2-d^2)^{1/2} - [m^2(1-th)^2 - d^2]^{1/2}\} \tag{6}$$

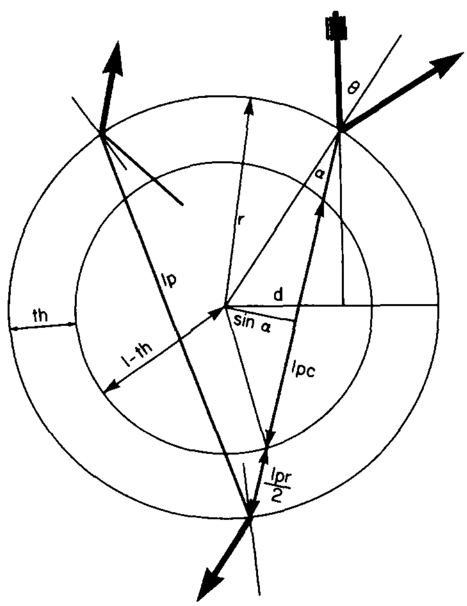

Fig. 1. Cross section of a ring-dyed fiber showing light paths through it and the angles of refraction and reflection.

where th is the thickness of the ring in which all the dye is concentrated.

The relation of the dye concentration in the ring cr, for a dye concentration c in the fiber as a whole is

$$cr = c/[th(2 - th)] \tag{7}$$

The fiber radius r in centimeters is obtained from the fiber denier and the fiber density fd,

$$r = (1/9 \times 10^5)^{1/2}(\text{denier}/fd)^{1/2} \tag{8}$$

From the weight concentration wc, the particle density pd, the particle diameter pe of the delustrant, and the fiber density fd, one obtains a uniform distance between the scattering particles dp,

$$dp = pe(pd/wc \cdot fd)^{1/3} \tag{9}$$

The cross-sectional area at the center of which the scattering particle is located is dp^2 and the cross-sectional area of the particle pe^2 or $\pi/4pe^2$ thus the fraction of light impinging on the scattering particle sa,

$$sa = pe^2/dp^2 \tag{10}$$

and the fraction unaffected by the particle

$$1 - sa = (dp^2 - pe^2)/dp^2 \tag{11}$$

The fraction e of light transmitted over the distance between scattering particles dp is

$$e = 10^{-[c/th(2-th)k \cdot dp]} \tag{12}$$

where k is the coefficient of extinction of the dye fiber system expressed for a lightpath 1 cm long.*

From the foregoing the lightpath lp is subdivided into a number of steps st from scattering particle to scattering particle. The number of those steps is

$$st = lp/dp \tag{13}$$

Since at each step sa fraction of the light interacts with a delustrant particle, the total amount transmitted through a fiber without scattering from first refraction to impingement on the fiber to air interface et is

$$et = (1 - sa)^{st} e^{(st+1)} \tag{14}$$

Since the total incident light intensity is I_0 then that of each bundle is $I_0/INCR$ of which $(I_0/INCR)(1 - \rho_\parallel)$ and $(I_0/INCR)(1 - \rho_\perp)$ enters the fiber and et times that amount is transmitted.

$\rho_\parallel$ and $\rho_\perp$ times the transmitted amount is internally reflected the first time, and et times that reaches the point of second internal reflection, and so on.

The amount of light which interacts with the first scattering particle encountered is I_{S1},

$$I_{S1} = (I_0/INCR)(1 - \rho_\parallel \text{ or } \rho_\perp) sa \cdot e^2 \tag{15}$$

and

$$I_{Sn} = (I_0/INCR)(1 - \rho_\parallel \text{ or } \rho_\perp)(1 - sa)^{(n-1)} sa e^n \tag{16}$$

interacts with the nth delustrant particle along the lightpath.

* In the earlier papers a quantity CK was used in which C was the dye concentration and K the coefficient of extinction expressed in terms of fiber radius.

If the lightpath includes the undyed core of the fiber, then e is, of course, equal to 1 in the core.

The angle of reflection is two times the angle of incidence and the angle refracted out of the fiber is

$$\text{refracted angle} = 2\theta + O(180 - 2\alpha) \tag{17}$$

when O is the order of refraction.

THE MODEL OF SCATTERING WITHIN THE FIBER

As indicated above it is assumed that the delustrant particles are uniformly distributed within the fiber. The reflection-refraction-absorption events are treated as in the previous papers except that at each particle the amount of light proceeding is reduced by the fraction of the cross-sectional area covered by that particle in the lightpath. It will be assumed that the fraction of light impinging on the scattering particle in this sequence of events is scattered in all directions (Fig. 2) while from all other particles radiation produced by this scattered light further scattering occurs only in the direction of that light and in the direction opposite to it. Without being able to produce proof it is assumed that the consequences of these further simplifications will cancel each other.

The light of intensity I_{sn} [eq. (16)] impinges on the nth scattering particle (Fig. 2) and the sf fraction of it is scattered uniformily in all directions and 1-sf is scattered in the direction in which the beam travels (Fig. 3). We will treat the first group of subsequent events as if all scattering resulted from the portion of light proceeding in the direction of the original beam, and that portion whose direction has been reversed will maintain that direction. It is clear that this light

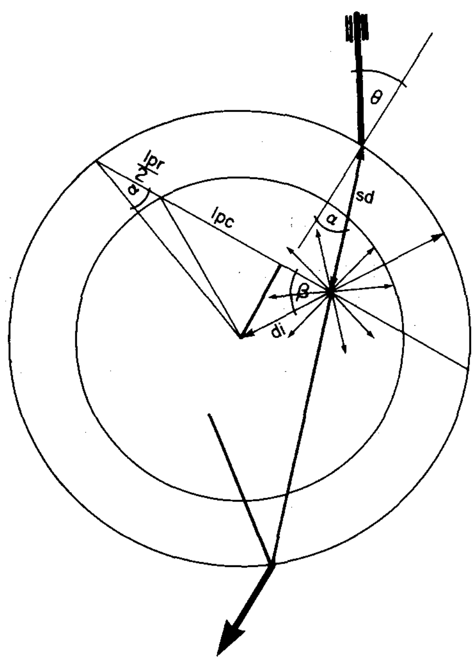

Fig. 2. Cross section of a fiber showing scattering around a particle in the light path considered.

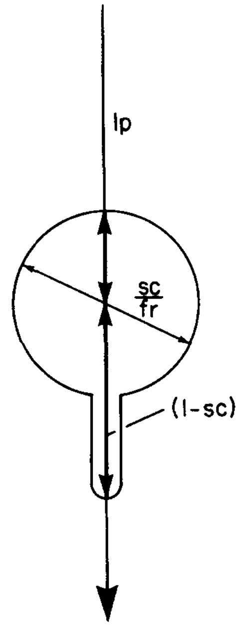

Fig. 3. Distribution of scattering intensity as assumed in model.

might travel part of its way through portions of the fiber containing dye (ring) and part of it through the portion devoid of dye (core). The intensity of the light which reaches the interfaces of fiber and air can then be calculated as follows:

The light scattered in the direction of the impinging beam is 1-*sc* that scattered in other directions sc/fr, where fr is the number of scattered directions which we choose to evaluate.

We will call n the number of delustrant particles in the original lightpath counting from the particle from which we compute to the opposite end of the lightpath, and m is the number of scattering particles from the first interface to the last particle before the one considered ($n + m$ is the number of particles in the lightpath). $R(n,m)$ is the fraction of light returned to the point of which that lightpath originates and $T(n,m)$ is the fraction which reaches the opposite interface (Fig. 4).

Through iteration from particle $m + 2$ to the end of the lightpath, $n + m$, and from m to the beginning of the lightpath, we get

$$R(y) = R(y-1) + T(y-1)^2e^2sa \cdot sc/[1 - R(y-1)e^2sa \cdot sc] \quad (18)$$

$$T(y) = T(y-1)e(1 - sa \cdot sc)/[1 - R(y-1)e^2sa \cdot sc] \quad (19)$$

when $R(1) = T(1) = 1/2$ we obtain the respective reflected and transmitted fractions of light. Combining these

$$R(x) = sa \cdot sc + R(n)^2e^2(1 - sa \cdot sc)^2/[1 - R(n)e^2sa \cdot sc] \quad (20)$$

$$T(x) = T(n)e(1 - sa \cdot sc)/[1 - R(n)e^2sa \cdot sc] \quad (21)$$

and finally

$$R(n+m) = T(m)R(x)e/[1 - R(m)R(x)e^2] \quad (22)$$

$$T(n+m) = T(x) + R(m)R(x)T(x)e^2/[1 - R(m)R(x)e^2] \quad (23)$$

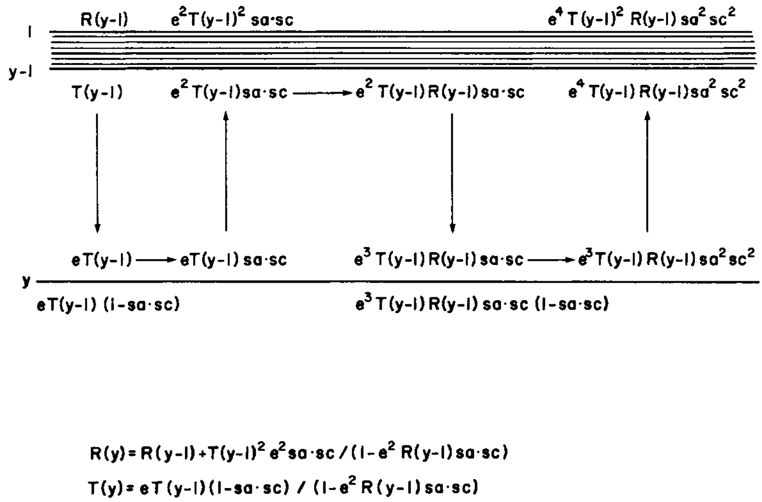

Fig. 4. Calculation of absorption and scattering through y particles.

These calculations have to be carried out for each scattering particle along the lightpath and at each scattering particle in directions other than that of this original lightpath. As mentioned before, along these "secondary" lightpaths we will consider only forward and backward scattering; thus, the coefficient fr, for these calculations will be set equal to 1.

All that remains is to calculate the number of scattering particles along the various secondary lightpath.

In Figure 5, p is one of the secondary lightpaths originating from scattering at the particle at the intersection of the original lightpath and the one in question. For purposes of calculation we will consider fr directions of scattering. The angle ϵ will then assume the values $1,2,\ldots, fr$ times $360/(fr + 2)$.

The values of lp, lpc, and lpr in terms of β are

$$lp = 2(r^2 - d^2 \sin^2\beta)^{1/2} \tag{24}$$

$$lpc = 2[r^2(1 - th)^2 - d^2 \sin^2\beta]^{1/2} \tag{25}$$

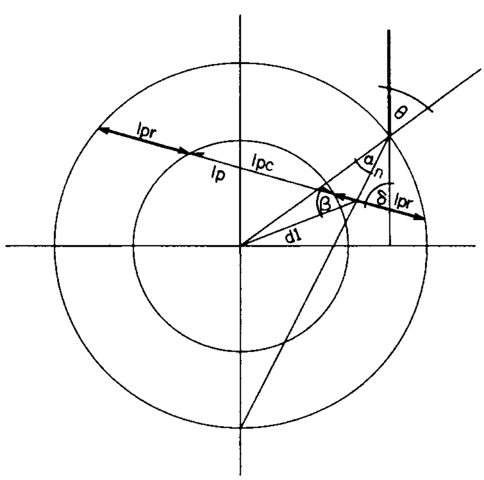

Fig. 5. One scattered light path.

$$lpr = lp - lpc \tag{26}$$

where

$$dl = \{r - n[1 - (d/m)^2]^{1/2}\}^2 + (nd/m)^2 \tag{27}$$

and

$$\beta = 180 - \alpha - \epsilon - \arcsin\,[(n/dl)d/m] \tag{28}$$

n is the number of the scattering particles from the fiber surface to the point of refraction of the primary beam.

The angle δ of the secondary lightpath in respect to the incident lightbeam is

$$\delta = 180 - \theta - \arcsin[(n/x)d/m] - \beta \tag{29}$$

The portion of the secondary lightpath between fiber surface and the point of scattering is for the left portion (in Fig. 4):

$$lp(\text{left}) = (r^2 - dl^2 \sin^2\beta)^{1/2} + x \cos\beta \tag{30}$$

and

$$lp(\text{right}) = lp - lp(\text{left}) \tag{31}$$

The angle α^1 of the scattered beam at the fiber-air interface is

$$\alpha^1 = \arcsin(dl \sin\beta \tag{32}$$

where dl and β are the quantities shown in Figure 5.

By Snell's law the angle of refraction is then

$$\text{angle} = \arcsin(mdl \sin\beta) \tag{33}$$

and with respect to the line perpendicular to the plane of the fabric

$$\text{angle} = \delta - \alpha^1 + \text{angle} \tag{34}$$

or

$$= \delta + \alpha^1 - \text{angle} \tag{35}$$

depending on whether it is to the left or right as represented in Figure 5.

THE LIGHTPATH THROUGH A PILE OF FABRIC

The fate of light impinging on the fiber surface is described substantially as in the paper by Allen and Goldfinger,[1] with the modifications resulting from the possibility of internal scattering. Since, as in the original model, we assume that neighboring fibers have no effect on each others lightpaths, we add all radiation reflected and refracted within the angles of 90° and 270° and call it "down" and that from 270° through 0° to 90° "up."

For two layers of fabric the reflection is given by

$$\text{up}(2) = \text{up}(1) + \text{up}(1)\,\text{down}(1)^2/[1 - \text{up}(1)^2] \tag{36}$$

and

$$\text{down}(2) = \text{down}(1)^2/[1 - \text{up}(1)^2] \tag{37}$$

and for n layers,

$$\text{up}(n) = \text{up}(n-1) + \text{down}(n-1)^2\text{up}(1)/[1 - \text{up}(n-1)\text{up}(1)] \tag{38}$$

and

$$\text{down}(n) = \text{down}(n-1)\text{down}(1)/[1 - \text{up}(n-1)\text{up}(1)] \tag{39}$$

RESULTS

The main results are summarized in Figures 6–8. Figure 6 gives reflectance curves for two fabrics, one made of internally scattering fibers and one of optically homogeneous fibers. Confirming the known observations, the fabric composed of internally scattering fibers reflects more light for the same amount of dye than the one made of optically homogeneous fibers.

A more sensitive test confirming the essential validity of the model is shown in Figures 7 and 8. Here the ratio of the reflectances of ring-dyed to homogeneously dyed fabrics are shown. In the case of homogeneous fibers we find a strong maximum of that ratio, lightening, as a result of ring-dyeing in the reflectance range of approximately 10%, as discussed in previous papers.[2–4] In the case of internal scattering this lightening effect is greatly reduced, and shifted to higher reflectances. In addition we observe a very strong minimum 'darkening' at lower reflectances.

Indeed it is known to dyers that internally scattering fibers are, under certain

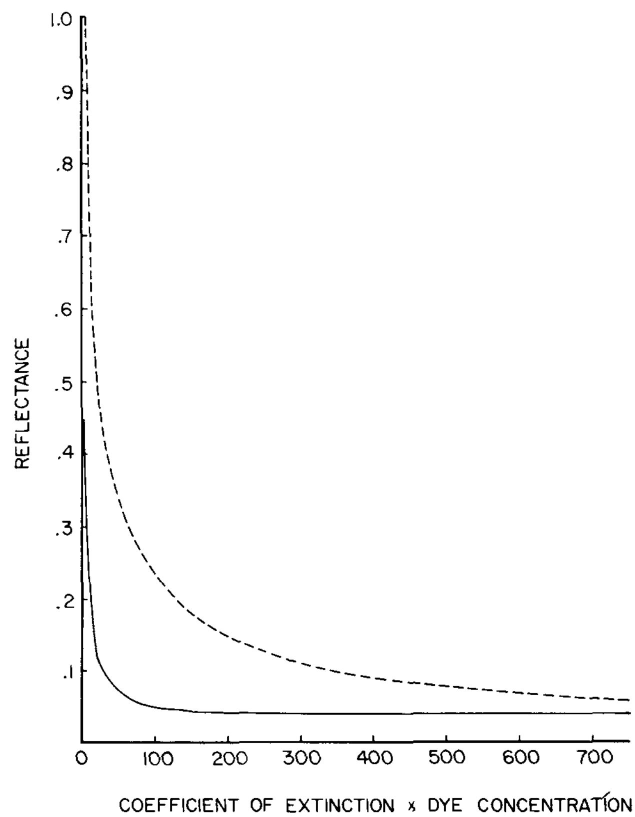

Fig. 6. Reflectance of a 20 denier, 1.4 density optically homogeneously dyed fiber of refractive index 1.6 (—) and of the same fiber with delustrant particles of 5×10^{-5} cm diam, density 4 at a concentration of 0.1 parts by weight (---) plotted against the product of the coefficient of extinction of the dye and its concentration.

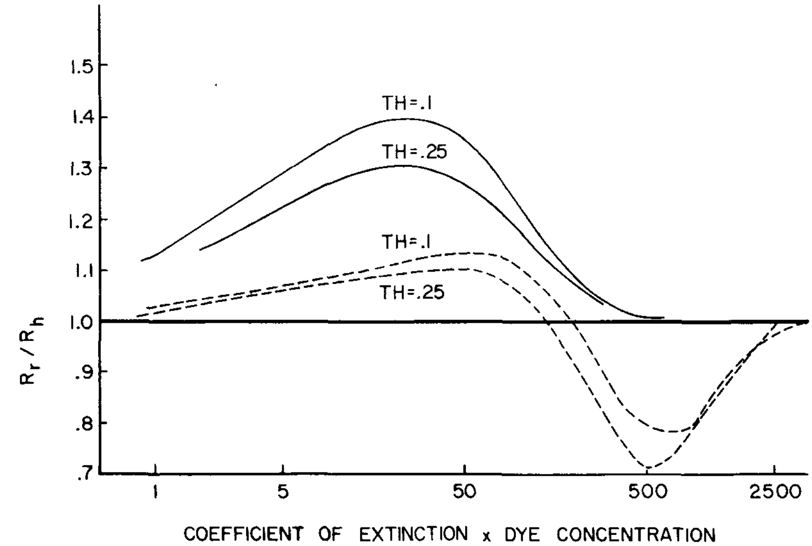

Fig. 7. Ratios of the reflectances of ring (R_r) and homogeneously (R_h) dyed fibers as described in caption to Figure 6 against the product of the coefficient of extinction of the dye and its concentration. The optically homogeneous fiber (—) and internally scattering fiber (···) reflectance ratios are given for ringthickenesses of 0.1 and 0.25 radii.

circumstances darker when the dye is concentrated in a ring, whereas in the case of optically clear fibers, ring distribution of the dye always leads to lightening.

It is implicit in this treatment that the presence of scattering particles, delustrants, will also have an effect on the hue of the dyed fabric. This is because of the dependence of the efficiency of scattering on the wavelength of light and the particle size and not as is sometime assumed by any absorption effects in the delustrant. This aspect will be dealt with in a future paper.

Comment: To save computer time, only the first refraction was considered and all secondary scattering was neglected in calculating the points of Figures 6–8. This additional simplification does change the position of the curves slightly but has no effect on their general shape.

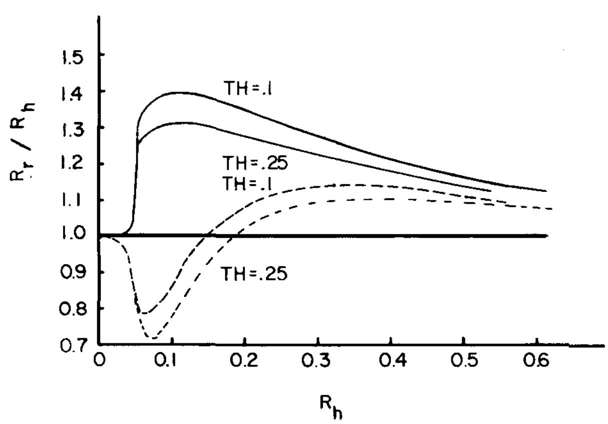

Fig. 8. Same ratios as in Figure 7 plotted against the reflectance of a homogeneously dyed fiber.

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